Use of Analog Models to Simulate Flow Recession of Karstic Springs

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Abstract

Analog models of reservoirs were developed from the analysis of flow recession for three karst springs in Lebanon. The first source is characterized by a linear relationship between flow and time; while the other two obey a power law. For the first source, the analog model is a reservoir with vertical walls, whereas the reservoirs for the other two consist of curved walls whose geometric equation is a power law with non-integer dimension. It is shown that for these two types of reservoirs, there is no linear relationship between the flow and the stored water volume at the same time, and that these analog models enable estimation of the stored water volume.

Keywords

Recession; Analog Models; Karstic Springs; Reservoir

Introduction

The recession analysis of flow springs permits to estimate the reserves of groundwater used to provide populations need, irrigation and industrial activities. The most frequent analysis method used is based on the linearity between the flow rate and the volume of water stored at the same time. The first published works on this subject are those of Boussinesq (Boussinesq, 1904) and Maillet (Maillet, 1905), and this linear model approach is still frequently used for karstic environment. To overcome the inability to simulate the recession of a spring flow by a single exponential law, the most common method used is to cut chronic discharge and fit, for each family of flow, an exponential law (Raesi, 2008). Some authors have questioned the linear property because of the inadequacy of the exponential law as a depletion model (Han and Hammond, 2006). Along with the physical approach of the phenomenon, some authors have developed methods for the analysis of probabilistic flow recession (Aksoy, 2004; Ceola et al, 2010). Probabilistic approaches have the advantage of

highlighting the Markovian character that is related or not to the possible effect of the karst memory but do not provide information on the geometric characteristics of the aquifer. These characteristics can be determined by the use of analog models (Carlier et al, 2012) even though the proposed model may not be unique (Carlier and Mroueh, 2012).

The objective of this paper is to propose a plausible recession model from the geometrical definition of analog reservoirs and identify available groundwater reserves.

Geological Setting of Karst Springs and Flood Recession Rates

Even though the values of flow recessions are few since they are done on monthly basis, they help highlighting the different modes of flow recession according to the spring.

Figures 1, 2 and 3 show that the flow recession, from May to September, obeys different laws. The decay low for the Rachiine river appears to be linear, whereas for the sources of Jouaiite and Abou Ali, the decay laws that give the best fit are power laws rather than exponential laws.

Recession Flow Analysis for the Source of Rachiine

Reservoir Model

The best fit of flow-time has been obtained by a linear relationship of the form:

$$Q(t) = b - a\,t \tag{1}$$

$b : (\mathrm{L^3 T^{-1}}); a : \mathrm{T^{-1}}$

Equation (1) is a mathematical model of recession and it is important to understand its physical reality. It has been shown (Carlier et al, 2012) that a linear law of flow
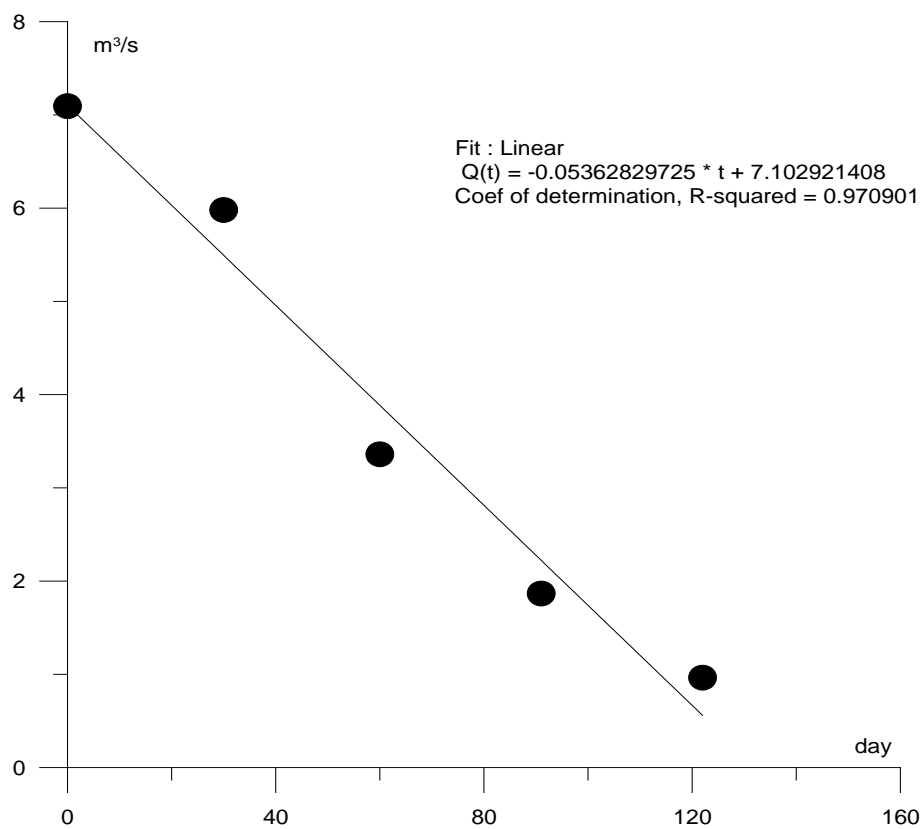
Fit : Linear
 Q(t) = -0.05362829725 * t + 7.102921408
Coef of determination, R-squared = 0.970901

FIG. 1 FLOW RECESSION FOR THE SOURCE OF RACHIINE

Fit 1: Exponential (red line)
 Q(t) = exp(-0.0221740491 * t) * 10.24949637
Coef of determination, R-squared = 0.986942

Fit 2: Power (blue line)
Q(t) = 571.4261153*t $^{-1.361721431}$
Coef of determination, R-squared = 0.995076

Fit 3: Power (dash line)
Q(t) = (0.00826*t+0.1404)$^{-1.78283}$
Coef of determination, R-squared = 0.999617

FIG. 2 FLOW RECESSION FOR THE SOURCE OF ABOU ALI

Fit 1: Exponential (red line)
Q(t) = exp(-0.03336216112 * t) * 1.676119022
Coef of determination, R-squared = 0.985589

Fit 2: Power (blue line)
Q(t) =154.9819401* t$^{-1.643021123}$
Coef of determination, R-squared = 0.99997

Fit 3: Power (dash line)
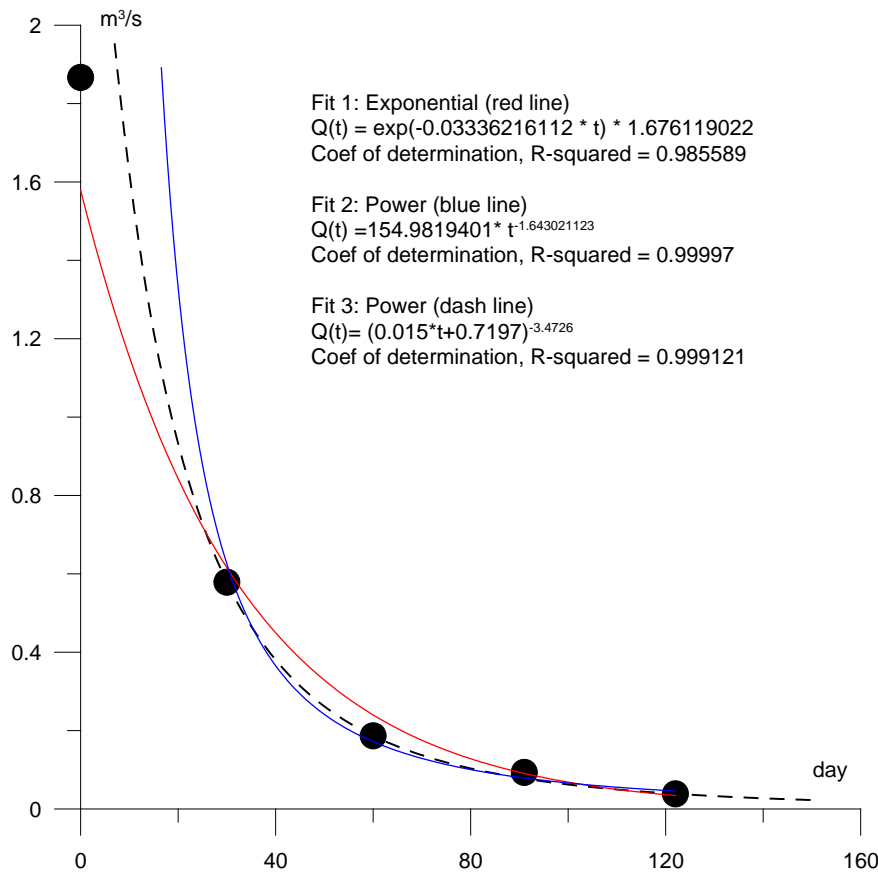Q(t)= (0.015*t+0.7197)$^{-3.4726}$
Coef of determination, R-squared = 0.999121

FIG. 3 FLOW RECESSION FOR THE SOURCE OF JOUAAIT

recessions in function of time corresponds to reservoirs with vertical walls (Figure 4).
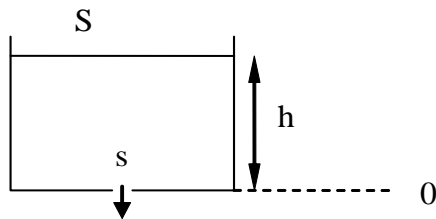

FIG. 4 RESERVOIR WITH VERTICAL WALLS

It has also been shown (Carlier et al, 2012) that by considering Bernoulli's law as a law of behavior (Equation 2) and combining it with the law of continuity in the absence of recharge (Equation 3), a decay law of the height h of water could be determined (equation 4) and that the parameters a and b of the calibration equation could be identified (equation 5).

$$Q(t) = K s [2 g h(t)]^{1/2} \qquad (2)$$

$$Q(t) = - S(t) [d h(t)/d t] \qquad (3)$$

K : loss coefficient, between 0 and 1, bonded to the irregular geometry and roughness of the outlet orifice of the reservoir

s : area of the outlet orifice of the reservoir (L^2)

g : acceleration due to gravity (L T^{-2})

$h(t)$: height of water above the orifice (L)

$S(t)$: Horizontal surface of the water at time t (L^2)

$$h(t) = (b - a t)^2 / (K^2 s^2 2 g) \qquad (4)$$

$$Q(t) = Q_0 - [(K^2 s^2 g)/S] t \qquad (5)$$

$$S = (K^2 s^2 g)/a \qquad (6)$$

Estimation of the Stored Water Volume

For this type of reservoir, it has been shown (Carlier et al, 2012) that the emptying time t_v is expressed in the following equation (7).

$$t_v = (2 S h_0^{1/2})/(K s (2 g)^{1/2}) = (2 S h_0)/(K s (2 g h_0)^{1/2}) = 2 V_0/Q_0 \quad (7)$$

V_0 is the volume of water stored in the reservoir at the start of the recession and Q_0 is the initial flow rate .

The emptying time can also be pulled from the equation (5) by considering $Q(t) = 0$:

$$t_v = (S Q_0)/(K^2 s^2 g) = b/a \qquad (8)$$

The relationship between the flow at time t and the volume of groundwater stored at the same time can be

established:

$$Q(t) = K s \left(2 g h(t)\right)^{1/2} = K s \left(2 g V(t) / S\right)^{1/2}$$
$$= A V(t)^{1/2} \tag{9}$$
$$A = K s \left(2 g / S\right)^{1/2}$$

Reservoirs with vertical walls are therefore called non-linear reservoirs. Attention should be paid to the terminologies since the non-linearity is related to the flow-volume relationship stored at the same time whereas the relationship between speed and time is perfectly linear.

The total stored water volume at the beginning of the recession is determined by the equation (9), for $t=0$ it is written as follow:

$$V_0 = \left(Q_0{}^2 S\right) / \left(2 g K^2 s^2\right) \tag{10}$$

By combining equations (6) and (10), the volume of water stored is:

$$V_0 = Q_0{}^2 / (2 a) = b^2 / (2 a) \tag{11}$$

The parameters a and b are determined from the statistical processing of field measurements.

Interpretation of the Recession and Its Validation

Recession obeys the following linear law:

$$Q(t) = 7.103 - 0.05363 \, t \tag{12}$$

Time t is per Day, the Flow Q is expressed in m^3 s^{-1}; therefore, the unit for parameter $a=0.05363$ is in m^{3}s^{-1} J^{-1}

By expressing the parameter a in m^3/s/s, its value will be modified to: $a= 6.207 \, 10^{-7}$ m^3/s/s

With: $a = K^2 s^2 g / S$

Emptying time in accordance to the equation (8), is equal to $t_v = 132.44$ days.

By setting arbitrary values for the coefficient of loss of charge K and for the section of the outlet s; the horizontal surface of the reservoir S can be calculated with the equation (6).

The initial height h_0 is calculated by equation (2), and the initial volume V_0 is equal to the product of $h_0 S$.

Table 1 gives some possible models of reservoirs with vertical walls displaying the same recession.

TABLE 1 PLAUSIBLE PHYSICAL MODELS FOR THE RECESSION

S m²	S m²	Q_0 m³/s	K	h_0 m	V_0 m³	EMPTYING TIME J
1	5689543	7.103	0.6	7.143	40640550	132.44
0.25	355596	7.103	0.6	114.28	40640550	132.44

point: measured data

line: computed data (HEC HMS)
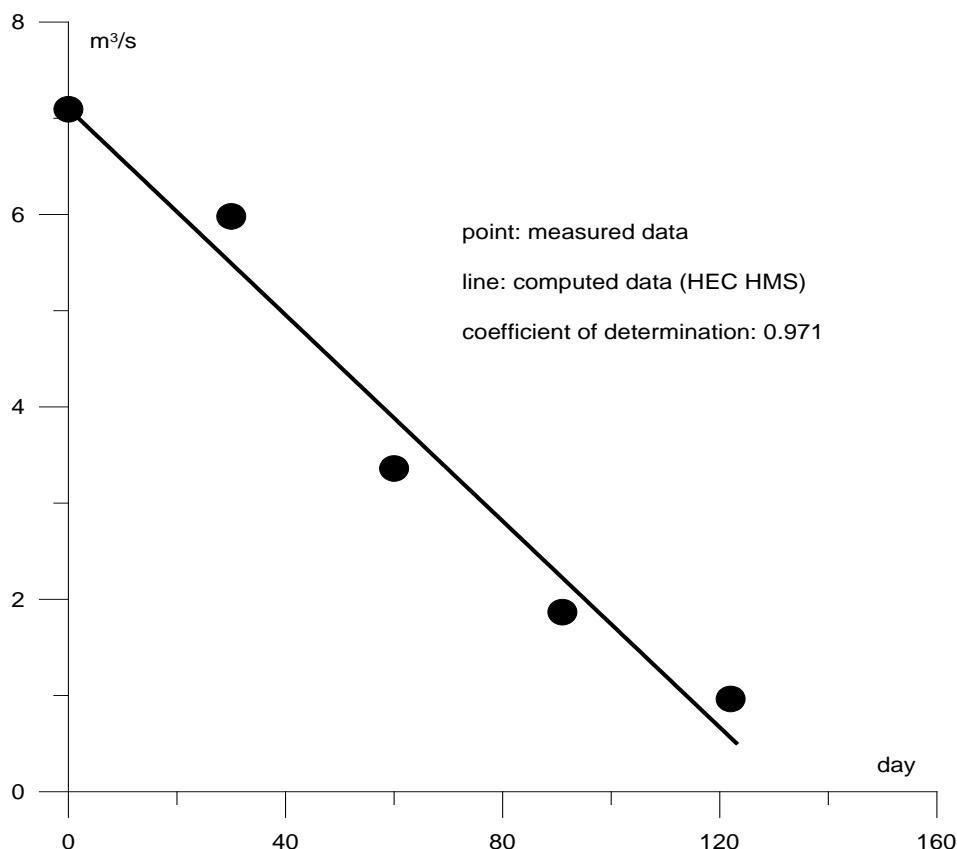
coefficient of determination: 0.971

FIG. 5 MODELING FLOW RECESSION WITH THE NUMERICAL CODE HEC-HMS

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The numerical code HEC-HMS (Hydrologic Modeling System), developed at the Hydrologic Engineering Center of the United States army, was used, with the purpose to model the drain of the reservoir with vertical walls having an outlet of 1 m², a coefficient of pressure loss of 0.6 and a constant horizontal surface of 5,689,543 m² with an initial water height of 7.14 m relative to the center of the outlet area of 1 m². The comparison between the calculated and measured flow is shown in Figure 5.

Analysis of ABOU ALI and JOUAIITE Flow Recession

Reservoir Model

Figures 2 and 3 display that the best fit is associated with the power laws. The power law is as follow:

$$Q = a t^{-\beta} \tag{13}$$

With $\beta > 0$, the initial condition $t=0$ states as a problem since the flow tends to go to infinity. In order to satisfy the initial condition $Q(t_0)=Q_0$, the following power law adopts:

$$Q = (a t + b)^{-\alpha} \tag{14}$$

with $b = Q_0^{-1/\alpha}$

It has been shown (Carlier et al, 2012) that the equation (14) corresponds to reservoirs with curved walls (Figure 6).

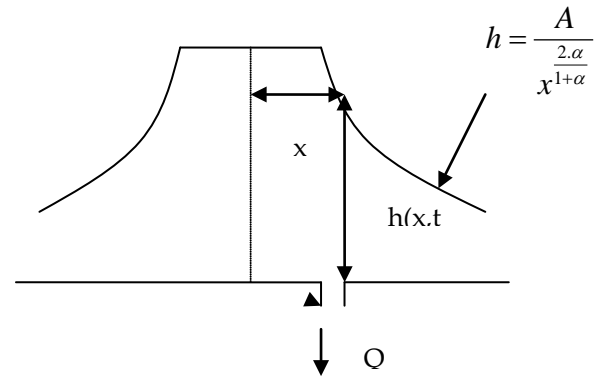$$h = \frac{A}{x^{\frac{2.\alpha}{1+\alpha}}}$$

FIG. 6 GEOMETRY OF THE RESERVOIR WITH CURVED WALLS

The equation (15) expresses the relation, for instant t, between the water level of the free surface h and the abscissa x:

$$h = A / x^{2\alpha/(1+\alpha)} \tag{15}$$

With:

$$A = 0.5 (2 a \alpha l)^{-2\alpha/(1+\alpha)} \left(K^2 s^2 g\right)^{(\alpha-1)/(\alpha+1)} \tag{16}$$

α : non integer dimension

Similarly

$$x = B / h^{(1+\alpha)/(2\alpha)} \tag{17}$$

With:

$$B = A^{(1+\beta)/(2\alpha)}$$
$$= 2^{-(3\alpha+1)/(2\alpha)} \left(K^2 s^2 g\right)^{(\alpha-1)/(2\alpha)} / (\alpha a l) \tag{18}$$

Equation (15) expresses the geometry of the wall tank.

TABLE 2 STORED WATER VOLUME

NAME	α	a with t in J	a with t in s	Q_0 CALCULATED m³/s	V_0 hm³	Q_0 MEASURED m³/s	V_0 hm³
ABOU ALI	1.78283	0.00826	9.5602E-08	33.1207	62.13	11.20	38.6
JOUAAITE	3.4726	0.015	1.7361E-07	3.1337	5.25	1.8668	3.63

TABLE 3 ESTIMATED PARAMETERS AND NON-INTEGER POWER

NAME	s (m²)	l (m)	K	α
ABOU ALI	2	500	0.6	1.78283
JOUAAITE	0.5	500	0.6	3.4726

TABLE 4 CALCULATED PARAMETERS AND INITIAL VALUES OF FLOW AND WATER LEVEL

NAME	A	B	Q_0 m³/s	h_0 (m)
ABOU ALI	70956.21	6108.998	33.1207	38.83
JOUAAITE	46647.57	1015.437	3.1337	5.56

TABLE 5 RECESSION EQUATIONS AND GEOMETRIC FUNCTIONS OF RESERVOIRS

NAME	EQUATION	$x=f(h)$
ABOU ALI	$Q = (0.00826t + 0.1404)^{-1.78283}$	$x = 6108.99799 / h^{0.78045299}$
JOUAAITE	$Q = (0.015t + 0.7197)^{-3.4726}$	$x = 1015.43651 / h^{0.64398433}$

Estimation of the Volume of Water Stored

The volume of water which has flowed out of the reservoir at time t is:

$$V_{out}(t) = \int_0^t Q(\tau)\,d\tau$$
$$= \left[(at+b)^{1-\alpha}\big/a(1-\alpha)\right] - \left[b^{1-\alpha}\big/a(1-\alpha)\right] \quad (19)$$

Equation (19) can be written with $b = Q_0^{-1/\alpha}$:

$$V_{out}(t) = \left[Q(t)^{\beta} - Q_0^{\beta}\right]\big/\left[a(1-\alpha)\right] \quad (20)$$

With $\beta = (\alpha-1)/\alpha$

For both sources, α is greater than 1, it can be deduced that for an infinitely large time t, the total volume of water restored is:

$$V_{out}(\infty) = -\left[b^{1-\alpha}\big/a(1-\alpha)\right] \quad (21)$$

Equation (21) is, in fact, the total volume of water stored at the beginning of the recession, or V_0.

It can be written that:

$$V_o = Q_0^{\beta}\big/\left[a(\alpha-1)\right] \quad (22)$$

It is deduced that the volume of water stored at time t is:

$$V(t) = V_0 - V_{out}(t) = Q(t)^{\beta}\big/\left[a(\alpha-1)\right] \quad (23)$$

With $\alpha > 1$ and $\beta > 0$

There is no linear relationship between flow and volume stored at the same time. This type of reservoir is called nonlinear.

Table 2 presents an estimation of available groundwater reserves.

Numerical Modeling

The initial rate calculated Q_0 is estimated by equation (14) for $t=0$.

By setting arbitrary values for the coefficient of loss K, the section of the outlet opening s and the width l of the reservoir, perpendicular to the x coordinate (abscissa), the parameters A and B are calculated by equations (16) and (18). The initial height of the water in the tank, h_0, is calculated by the equation (2).

Equation (17) permits the calculation of the corresponding x_0 values and its homologue h_0.

Table 3, 4 and 5 summarize the estimated and calculated values that have helped define the geometry of the tanks.

Once the geometry of the reservoir has been defined, and introduced into the code HEC-HMS, the simulation of the emptying of the reservoir can be simulated. The results are shown in Fig (7) and (8).
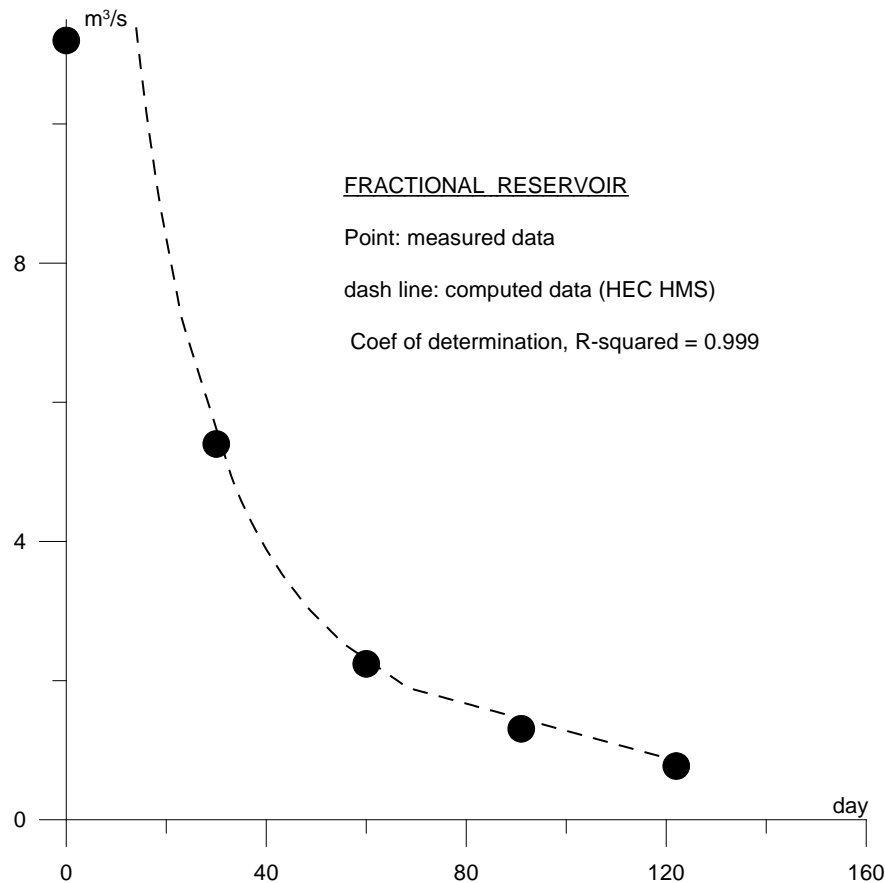

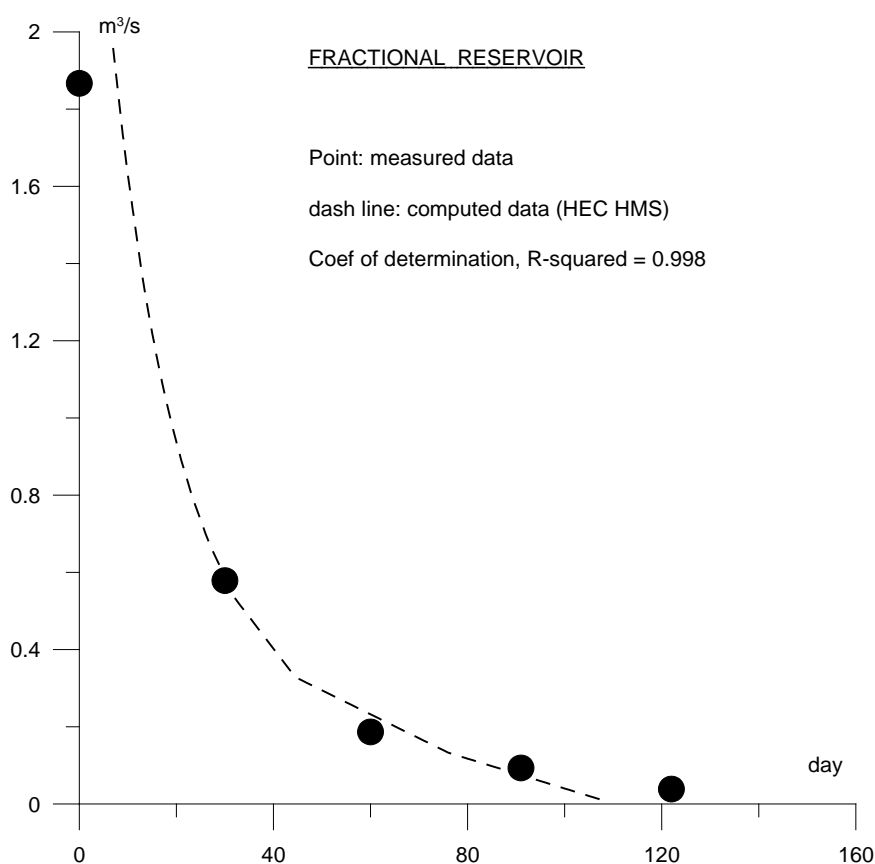FIG. 7 ABOU ALI. RECESSION ACCORDING TO A NON-INTEGER DIMENSION TANK

FIG. 8 JOUAIITE RECESSION ACCORDING TO A NON-INTEGER DIMENSION TANK

Conclusion

The analog models presented have a descriptive function. The proposed approach shows a close relationship between the shape of the flow recession curves and geometry of analog reservoirs. It also shows that even if there is no unique model for a given analogue chronic flow, reservoirs belong to the same geometric family. Following this analysis, it appears that the modeling of flow recessions by exponential law is very questionable, even objectionable for karst environments. Indeed, the exponential law for recession implies a linear relationship between flow and volume stored at the same time, yet the models developed for the three Lebanese sources are highly nonlinear. This prospective analysis of flow opens up new interpretations that should be tested systematically on the karstic sources.

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